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The role of $K_0^*(1430)$ in $D \to PK$ and $\tau \to KP\nu_\tau$ decays

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ABSTRACT

We consider the scalar form factor in the weak current matrix element $< PK|j_\mu|0 >$, $P = \pi, \eta, \eta'$. It obtains the contributions from the scalar meson resonance $K_0^*(1430)$ and from the scalar projection of the vector meson $K^*(892)$ resonance. We analyze decay amplitudes of the Cabibbo suppressed decays $D \to KP$, $P = \pi, \eta, \eta'$ using the factorization approach. The form factors of the relevant matrix elements are described by assuming the dominance of nearby resonances. The annihilation contribution in these decays arises from the matrix element $< PK|j_\mu|0 >$. All the required parameters are experimentally known except the scalar meson $K_0^*(1430)$ decay constant. We fit the decay amplitudes and we find that final state interaction improves the agreement with the experimental data. Then we extract bounds on scalar form factor parameters and compare them with the experimental data obtained in the analyses of $K \to \pi e \nu_e$ and $K \to \pi \mu \nu_\mu$. The same scalar form factor is present in the $\tau \to KP\nu_\tau$ decay, with $P = \pi, \eta, \eta'$. Using the obtained bounds we investigate the significance of the scalar meson form factor in the $\tau \to KP\nu_\tau$, $P = \pi, \eta, \eta'$ decay rates and spectra. We find that the $K_0^*(1430)$ scalar meson dominates in the $\tau \to K\eta'\nu_\tau$ decay spectrum.

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1 Introduction

The matrix element of the weak current in the K_{l3} decays is usually described by the use of vector and scalar form factors [1]

$$< \pi^0(p)|\bar{s}\gamma_\mu(1-\gamma_5)u|K^+(p') >= F_+(q^2)\left(p'_\mu + p_\mu - \frac{m_K^2 - m_\pi^2}{q^2}(p'_\mu - p_\mu)\right) + \\ + F_0(q^2)\frac{m_K^2 - m_\pi^2}{q^2}(p'_\mu - p_\mu). \tag{1}$$

Analyses of K_{l3} data assume a linear dependence on $q^2 = (p' - p)^2$:

$$F_{+,0}^{K,\pi}(q^2) = F_{+,0}^{K,\pi}(0)(1 + \lambda_{+,0}\frac{q^2}{m_\pi^2}), \tag{2}$$

with the experimental fit for λ_+ compatible with $\lambda_+ \sim 0.03$ both from K_{e3} and $K_{\mu 3}$ decays [1]. The data for λ_0 are rather inconclusive as summarized in [1], with $\lambda_0 = 0.006 \pm 0.007$ obtained from $K_{\mu 3}^+$ decays, while $K_{\mu 3}^0$ decay experiments prefer $\lambda_0 = 0.025 \pm 0.006$. The calculation done within chiral perturbation theory (CHPT) found $\lambda_0 = 0.017 \pm 0.004$ [2].

The same matrix element $< PK|j_\mu(0)|0 >$, $P = \pi, \eta, \eta'$ might appear as one of the contributions in some of the D meson nonleptonic decay amplitudes [3, 4], as well as in the $\tau \to KP\nu_\tau$ decays.

A satisfactory explanation of the mechanism of exclusive nonleptonic weak D meson decays has not been found yet. The method of heavy quark expansion cannot be used successfully on charm sector due to too small c quark mass. The simple widely used factorization ansatz for the matrix elements of the amplitude does not explain properly the Cabibbo allowed D^0 decays [3]-[8]. There are some attempts to approach the nonfactorized contribution [9]-[14]. However, the factorization technique [3, 4, 5, 6] is mostly used to treat the D meson decay amplitudes. Using this approach the amplitude of the D meson nonleptonic decay is divided into well known *spectator* and *annihilation* contribution [3, 4, 5]. Contrary to the quite well understood spectator contribution the annihilation contribution is rather poorly known.

In [4] an attempt has been made to treat the D meson nonleptonic decays assuming that the final state interaction (FSI) is dominated by nearby resonances. In this approach the annihilating contribution was fitted directly and the authors of [4] noticed that this contribution is larger than one would get by assuming the $K_0^*(1430)$ dominance of the scalar form factor with parameters fixed from $K \to \pi l \nu_l$ decays [3].

Motivated by this attempt we reinvestigate the $D^0 \to \bar{K}^0 P$, $P = \pi^0, \eta, \eta'$ and $D^0 \to K^-\pi^+$ decays in which the annihilation contribution is proportional to the scalar form factor of the matrix element $< PK|j_\mu|0 >$. We assume the $K_0^*(1430)$

meson dominance of the scalar form factor and we include the scalar projection of the vector meson $K^*(892)$ resonances following the idea of [15].

In our numerical calculation we use all existing data on $\tau \to K^*(892)\nu_\tau$, $K^*(892) \to K\pi$, $K_0^*(1430) \to K\pi$. Unfortunately, the present experimental data on K_{l3} decay [1] cannot give reliable bounds on the scalar meson decay constant. Therefore, we make fit using the decay rates of the $D^0 \to \bar{K}^0 P$, $P = \pi^0, \eta, \eta'$ and $D^0 \to K^-\pi^+$ data. We find better agreement with the experimental data if additional final state interaction is taken into account. In treatment of FSI we follow the idea of [4] and then extract bounds on the $K_0^*(1430)$ decay constant. Making the low-energy expansion of our scalar form factor, we then compare our result with the K_{l3} data and the chiral perturbation theory result.

The scalar form factor in the $\tau \to K\pi\nu_\tau$ decay has motivated a number of studies [15, 16, 17]. In [15] the scalar form factor was approached by exchange of resonance $K_0^*(1430)$ noticing the presence of the scalar projection of off-shell vector resonances $K^*(892)$. Using bounds on the scalar form factor in the $< PK|j_\mu|0 >$ matrix element, determined from the study of the D nonleptonic decays, we calculate the $\tau \to KP\nu_\tau$, $P = \pi, \eta, \eta'$, decay spectra and widths. We notice that the scalar meson contribution is rather small in $\tau \to KPl\nu_l$, $P = \pi, \eta$, while it gives the dominant contribution in the $\tau \to K\eta'\nu_\tau$ decay rate.

In Sec. 2 we reinvestigate the annihilation contribution of the $D^0 \to KP$, $P = \pi, \eta, \eta'$ decays. In Sec. 3 we analyze the effects of the final state interaction in these decay amplitudes. Sec. 4 is devoted to τ decays. Short summary of our results is given in Sec. 5.

2 Annihilation contribution in the D meson nonleptonic decays

We briefly review the use of factorization approximation in the weak nonleptonic D meson decays [3, 4, 5, 6, 7, 8, 18]. The effective weak Hamiltonian for nonleptonic decays of charmed particles is given by

$$H_{\text{eff}}^w = \frac{G_f}{\sqrt{2}}(\sum_{q_1 q_2} V_{cq_1}^* V_{uq_2}[a_1(\bar{q}_1 c)^\mu(\bar{u}q_2)_\mu + a_2(\bar{q}_1 q_2)_\mu(\bar{u}c)^\mu] + h.c.), \tag{3}$$

where $q_{1,2}$ stand for s or d quark operators, while currents $\bar{q}_1\gamma_\mu(1-\gamma_5)q_2$ are abbreviated as $(\bar{q}_1 q_2)_\mu$ and $V_{qq'}$ are the CKM matrix elements. In the factorization approximation approach the invariant amplitude is written in terms of current

matrix elements between vacuum and meson states

$$
\begin{aligned}
M^W_{P_1P_2,D} = \\
&\frac{G_f}{\sqrt{2}}\sum_{q_1q_2} V^*_{cq_1}V_{uq_2}[a_1\Big(\langle P_2|(\bar{u}q_2)_\mu|0\rangle\langle P_1|(\bar{q_1}c)^\mu|D\rangle + \langle P_1|(\bar{u}q_2)_\mu|0\rangle\langle P_2|(\bar{q_1}c)^\mu|D\rangle + \\
&\langle P_1P_2|(\bar{u}q_2)_\mu|0\rangle\langle 0|(\bar{q_1}c)^\mu|D\rangle\Big) + a_2\Big(\langle P_2|(\bar{q_1}q_2)_\mu|0\rangle\langle P_1|(\bar{u}c)^\mu|D\rangle + \\
&\langle P_1|(\bar{q_1}q_2)_\mu|0\rangle\langle P_2|(\bar{u}c)^\mu|D\rangle + \langle P_1P_2|(\bar{q_1}q_2)_\mu|0\rangle\langle 0|(\bar{u}c)^\mu|D\rangle\Big)].
\end{aligned}
\tag{4}
$$

Here $a_1 = 1.26 \pm 0.04$ and $a_2 = -0.51 \pm 0.05$ are effective Wilson coefficients [3].

Through Lorentz covariance we can write the most general expressions for matrix elements

$$
\begin{aligned}
\langle P_1(p_1)|\, V^\mu\, |P_2(p_2)\rangle =& F_+^{(P_2\to P_1)}(q^2)\left(p_1^\mu + p_2^\mu - \frac{m_2^2 - m_1^2}{q^2}q^\mu\right) + \\
&+ F_0^{(P_2\to P_1)}(q^2)\frac{m_2^2 - m_1^2}{q^2}q^\mu,
\end{aligned}
\tag{5}
$$

$$
\langle 0|\, A^\mu\, |P(p)\rangle = i\, f_P p^\mu, \tag{6}
$$

$$
\langle 0|\, V^\mu\, |S(p)\rangle = f_S p^\mu, \tag{7}
$$

$$
\langle 0|\, V^\mu\, |R(p,\lambda)\rangle = iF_R\epsilon^\mu(p,\lambda), \tag{8}
$$

where V^μ and A^μ are vector and axial currents, $g^\mu = p_1^\mu - p_2^\mu$, while P, S, R denote pseudoscalar, scalar and vector mesons respectively. Defining $\langle P_1P_2|(\bar{q}_1' q_2)_\mu|0\rangle = J^{12}_\mu$ one obtains from (5) [15, 19]

$$
\begin{aligned}
J^{12\mu} =& F_+^{(P_2\to P_1)}(q^2)\left(p_1^\mu - p_2^\mu - \frac{m_1^2 - m_2^2}{q^2}(p_1^\mu + p_2^\mu)\right) + \\
&+ F_0^{(P_2\to P_1)}(q^2)\frac{m_1^2 - m_2^2}{q^2}(p_1^\mu + p_2^\mu).
\end{aligned}
\tag{9}
$$

To evaluate form factors $F_{0,+}^{(P_2\to P_1)}(q^2)$ in (9) one usually uses single pole approximation [3, 17, 18, 20] from which one concludes that annihilation term contribution to decay width is negligible. It was noticed in [4] that this is not the case for two particle nonleptonic D meson decays, where final state has $S \neq 0$. This contribution was included in [4] by expressing the annihilation current densities $\partial^\mu J^{12}_\mu$ in terms of unknown variables multiplied by the $SU(3)$ Clebsch-Gordan coefficients. The magnitude of the coefficients turns out to be considerably larger

than what one would obtain assuming the single pole approximation with parameters fixed from $K \to \pi l \nu_l$ decays [3]. Here we reinvestigate the size of the annihilating contribution by assuming the meson dominance of the $\partial^\mu J^{12}_\mu$ matrix element.

In the analysis [15] of the $\tau \to K\pi\nu_\tau$ decay the scalar form factor received the contribution from the scalar projection of the off-shell vector resonances and from the scalar resonance $K_0^*(1430)$. In our present approach we include this contribution of $K^*(892)$ and the contribution of the $K_0^*(1430)$ scalar meson.

In order to use the existing experimental data, we use following properties of the resonances. The $SU(3)$ symmetric effective strong Hamiltonian describing the decays of $K^*(892) \to KP$, and $K_0^*(1430) \to KP$, with $P = \pi, \eta, \eta'$ is given by

$$H^S_{\rm eff} = i g_V \,{\rm Tr}([\partial_\mu \Pi, \Pi] V^\mu) + g_S \,{\rm Tr}(\Pi\Pi S), \tag{10}$$

where Π, V, S are 3×3 matrices

$$\Pi = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} + \frac{\eta_0}{\sqrt{3}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} + \frac{\eta_0}{\sqrt{3}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_0 \end{pmatrix}, \tag{11}$$

while S and V have corresponding scalar or vector mesons as matrix elements. The $SU(3)$ flavor symmetry breaking is taken into account through the physical masses and decay widths. The coupling constants g_S and g_V are some yet unknown coefficients that describe the strength of strong interaction. In order to account as much as possible for the $SU(3)$ symmetry breaking effects we calculate relevant parameters directly from the corresponding decays $K^*(892)$, $K_0^*(1430)$ and $\tau \to K^*\nu_\tau$, emphasizing this through the change of notation $g_V \to g_V(K^*)$, $g_S \to g_S(K_0^*)$. In our analysis we did not include the vector meson $K^*(1410)$ which presence was mentioned in [15] due to inducement of the large uncertainty in the vector form factor of the $\partial^\mu J^{12}_\mu$. Following [15] we obtain the current (9) for the intermediate $K^*(892)$ and $K_0^*(1430)$ meson states

$$\begin{aligned} J^{12}_\mu =& g_V(K^*) 2 a_{12K^*} F_{K^*} \frac{g_{\mu\nu} - \frac{q_\mu q_\nu}{m^2_{K^*}}}{q^2 - m^2_{K^*} + i\sqrt{q^2}\Gamma_{K^*}} (p_1^\nu - p_2^\nu) + \\ & f_{K_0^*} g_S(K_0^*) \frac{c_{12K_0^*} q^\mu}{q^2 - m^2_{K_0^*} + i\sqrt{q^2}\Gamma_{K_0^*}}, \end{aligned} \tag{12}$$

where $q = p_1 + p_2$, while

$$2a_{12K^*}(p_1 - p_2)_\mu \epsilon^\mu = \langle P_1 P_2 | \,{\rm Tr}([\partial_\mu \Pi, \Pi] V^\mu) | K^* \rangle, \tag{13}$$

$$c_{12K_0^*} = \langle P_1 P_2 | \,{\rm Tr}(\Pi\Pi S)) | K_0^* \rangle. \tag{14}$$

The unknown coupling constant F_{K^*} can be determined from the measured decay rate $\tau \to K^{*-}(892)\nu_\tau$, while $f_{K_0^*}$ is left as a free parameter.

The decay widths $\Gamma_X(q^2)$ in (12) are taken to be energy dependent [21]

$$\Gamma_X(q^2) = \Gamma_X \frac{m_X^2}{q^2}\Big(\frac{p(q^2)}{p(m_X^2)}\Big)^{2n+1}, \tag{15}$$

$$p(q^2) = \frac{1}{2\sqrt{q^2}}\sqrt{[q^2-(m_{P_1}+m_{P_2})^2][q^2-(m_{P_1}-m_{P_2})^2]}, \tag{16}$$

where $n = 1$ for the $K^*(892)$ (p-wave phase space) and $n = 0$ for the K_0^* (s-wave phase space)

The following expressions for form factors are obtained by matching (12) with (9)

$$F_+^{(P_2\to P_1)}(q^2) = \frac{g_V(K^*)2a_{12K^*}F_{K^*}}{q^2 - m_{K^*}^2 + i\sqrt{q^2}\Gamma_{K^*}}, \tag{17}$$

$$F_0^{(P_2\to P_1)}(q^2) = \frac{g_V(K^*)2a_{12K^*}F_{K^*}(1-q^2/m_{K^*}^2)}{(q^2 - m_{K^*}^2 + i\sqrt{q^2}\Gamma_{K^*})} + \frac{q^2}{(m_{P_1}^2 - m_{P_2}^2)}\frac{f_{K_0^*}g_S(K_0^*)c_{12K_0^*}}{(q^2 - m_{K_0^*}^2 + i\sqrt{q^2}\Gamma_{K_0^*})}. \tag{18}$$

Note that in (12) as suggested by [15] we used a $K^*(892)$ propagator proportional to $(g_{\mu\nu} - q_\mu q_\nu/m_{K^*}^2)$ and not, as sometimes suggested, proportional to $(g_{\mu\nu} - q_\mu q_\nu/q^2)$. The latter form would have led to a vanishing contribution of the vector resonance to the scalar form factor. However, this choice of propagator would fail to fulfill the condition $F_+(0) = F_0(0)$ and would thus lead to an unphysical divergence in current (9) as $q^2 \to 0$. The contribution of vector meson to the scalar form factor on the other hand does vanish on the $K^*(892)$ mass-shell when $q^2 = m_{K^*}^2$, this being in accordance with the expectations from dispersion relations [22].

The $K^*(892)$ meson decays almost exclusively into $K\pi$ final state [1]. Therefore we find using (10)

$$\Gamma(K^*(892) \to K\pi) = \frac{g_V^2(K^*)p_{K\pi}^3}{4\pi m_{K^*}^2} \tag{19}$$

with rest frame momentum $p_{K\pi}$ of outgoing K or π meson

$$p_{K\pi}^2 = \frac{m_{K^*}^2}{4}[(1-\frac{(m_K+m_\pi)^2}{m_{K^*}^2})(1-\frac{(m_K-m_\pi)^2}{m_{K^*}^2})]. \tag{20}$$

The value $\Gamma_{K^*} = 50.8$ MeV [1] yields $g_V(K^*) = 4.59$.

The value of F_{K^*} is extracted from $\tau \to K^*(892)\nu_\tau$ decay width $\Gamma_{\tau\to K^*\nu_\tau} = 2.905 \cdot 10^{-14}$ GeV [1]. Using

$$\Gamma_{\tau\to K^*(892)\nu_\tau} = \frac{1}{4\pi m_\tau}G_f^2 F_{K^*}^2(2+\frac{m_\tau^2}{m_{K^*}^2})p^2\sin^2\Theta_C \tag{21}$$

with $p = (m_\tau^2 - m_{K^*}^2)/(2m_\tau)$ we find $F_{K^*} = 0.195\,\mathrm{GeV}^2$.

In order to fix the $g_S(K_0^*)$ parameter we use the total decay width $\Gamma(K_0^*(1430)) = 287 \pm 23\,\mathrm{MeV}$ and branching ratio $B(K_0^*(1430) \to K\pi) = 93 \pm 10\%$ [1]. Assuming that $K_0^*(1430)$ dominantly decays into KP, with $P = \pi, \eta$, instead of using nonet symmetry argument we allow in (10) that scalar and psuedoscalar meson singlets have different couplings than octet states [4]. In order to implement the $\eta - \eta'$ mixing, we shall use the two-mixing angle formalism of the decay constants proposed by [23], extended to the mixing of the η, η' states through q^2 dependent mixing angle in [24]. Thus one has $|\eta >= \cos\theta_\eta |\eta_8 > - \sin\theta_\eta |\eta_0 >$ and $|\eta' >= \sin\theta_{\eta'} |\eta_8 > + \cos\theta_{\eta'} |\eta_0 >$. The mixing angles were found to be $\theta_\eta = -6.5^\circ \pm 2.5^\circ$ and $\theta_{\eta'} = -23.1^\circ \pm 3^\circ$ [24].

The effective Hamiltonian describing the deviation from the nonet symmetry is given by

$$\mathcal{H}_{\rm eff}^S = g_{888}\,\mathrm{Tr}\left(\Pi_8^\dagger \Pi_8^\dagger \mathrm{S}_8\right) + \frac{1}{\sqrt{3}} g_{818}\,\mathrm{Tr}\left((\eta_0^\dagger \Pi_8^\dagger + \Pi_8^\dagger \eta_0^\dagger)\mathrm{S}_8\right), \tag{22}$$

where $\Pi_8 = \Pi - \frac{1}{3}\mathrm{Tr}(\Pi) I$ and $\mathrm{S}_8 = \mathrm{S} - \frac{1}{3}\mathrm{Tr}(\mathrm{S}) I$. The exact nonet symmetry would require $g_S(K_0^*) = g_{888} = g_{818}$.

For g_{888} we calculate $|g_{888}| = |g_S(K_0^*)| = 3.67 \pm 0.3$ GeV from the partial decay width $\Gamma(K_0^*(1430) \to K\pi)$. In order to fix the value g_{888}/g_{818} we notice that the experimental errors might bring rather large deviation from the nonet symmetry. However, we find that the nonet symmetry solution for $g_{888}/g_{818} = 1$ is the average value of the allowed range and will use it from now on.

It is rather difficult to obtain the reliable estimate of the $f_{K_0^*}$. There are many different values in the literature depending on the model assumption. The QCD sum rule estimate in the K_0^* narrow width approximation thus gives $f_{K_0^*} \simeq 31 \pm 3$ MeV [25], the pole dominance result in $f_{K_0^*} \sim 50$ MeV [20], effective Lagrangian including width corrections estimate $f_{K_0^*} \sim 45\,\mathrm{MeV}$. The extraction from the decay rate $B(D^+ \to \pi^+ \bar{K}_0^{*0})$ using the factorization approach results in $f_{K_0^*} = 0.293 \pm 0.020$ GeV. Some estimates [26], [27] gave $f_{K_0^*}/f_{a_0} \sim 24 - 30$, while from $\tau \to \eta\pi\nu_\tau$ there is an upper limit $f_{a_0} < 7$ MeV [17].

Having fixed all parameters required by (18), except the decay constant of the $K_0^*(1430)$, we try to estimate its size using the K_{l3} data. Making low energy expansion of (18) we find

$$F_+^{21}(q^2) = -g_V(K^*) \frac{2a_{12K^*} F_{K^*}}{m_{K^*}^2} (1 + \frac{q^2}{m_{K^*}^2} + \ldots), \tag{23}$$

$$F_0^{21}(q^2) = -g_V(K^*) \frac{2a_{12K^*} F_{K^*}}{m_{K^*}^2} (1 + \frac{q^2}{(m_1^2 - m_2^2)} \frac{f_{K_0^*} g_S(K_0^*) c_{12K_0^*}}{F_{K^*} g_V(K^*) 2a_{12K^*}} \frac{m_{K^*}^2}{m_{K_0^*}^2} + \ldots). \tag{24}$$

Comparing it with the K_{l3} result in (2) we obtain

$$\frac{1}{(m_1^2 - m_2^2)} \frac{f_{K_0^*} g_S(K_0^*)}{F_{K^*} g_V(K^*)} \frac{m_{K^*}^2}{m_{K_0^*}^2} = \lambda_0 \frac{1}{m_\pi^2}. \tag{25}$$

In the case of $K_{\mu 3}^+$ $\lambda_0 = 0.006 \pm 0.007$ from which follows $f_{K_0^*} = 0.061 \pm 0.07\,\text{GeV}$ in fairly good agreement with theoretical values cited above, while in the case of $K_{\mu 3}^0$ $\lambda_0 = 0.025 \pm 0.006$ from which $f_{K_0^*} = 0.245 \pm 0.06$ GeV. Due to the fairly large discrepancy between these numbers, we shall use $f_{K_0^*}$ as a free parameter in our further analysis.

In case of λ_+ there are no such experimental ambiguities. The theoretical value for

$$\lambda_+ = \frac{m_\pi^2}{m_{K^*}^2}, \tag{26}$$

$\lambda_+ = 0.0243$ is in fairly good agreement with experimental data, which concentrate around $\lambda_+ \sim 0.03$ [1].

We analyze final states with $S \neq 0$. Measured are the decay rates for $D^0 \to \pi^0 \bar{K}^0$, $D^0 \to \pi^+ K^-$, $D^0 \to \eta \bar{K}^0$, $D^0 \to \eta' \bar{K}^0$, $D^0 \to K^+ \pi^-$, while for the $D_s^+ \to K^0 \pi^+$ an experimental upper bound exists.

We calculate invariant amplitudes using factorization approximation and form factors $F_{1,0}^{K\to\pi}$ in (24). The invariant amplitude is thus

$$\begin{aligned} M_{D\to P_1 P_2} = \frac{G_f}{\sqrt{2}} \Bigg[& C_{P_1}^{D\to P_1 P_2} i f_{P_2} F_0^{D\to P_1}(m_{P_2}^2) \left(m_D^2 - m_{P_1}^2 \right) + \\ & C_R^{D\to P_1 P_2} i f_D f_R \frac{m_D^2}{m_D^2 - m_R^2} \langle P_1 P_2 | H_{\text{eff}}^S | R \rangle \Bigg], \end{aligned} \tag{27}$$

where R denotes the intermediate resonance, while $C_{P_1}^{D\to P_1 P_2}$ and $C_R^{D\to P_1 P_2}$ are coefficients presented in Table 1.

In the D meson decay modes with the neutral K mesons in final state one has to notice that in experiment one sees a short lived K_S [28], ($K_{S,L} = 1/\sqrt{2}(K^0 \mp \bar{K}^0)$). In the Cabbibo allowed decays of the type $D \to \bar{K}^0 + X$ the annihilation contribution obtains the additional contribution due to the interference of the Cabbibo allowed $D \to \bar{K}^0 + X$ and Cabbibo doubly suppressed $D \to K^0 + X$ decay amplitudes that has not been taken into account in the analyses of experimental data. Thus we make a fit to the experimental data Γ_{PDG} [1] knowing that the measured decay width corresponds to K_S as

$$\Gamma_{\text{Exp}}(D \to K_S X) = \frac{1}{2} \Gamma_{\text{PDG}}(D \to \bar{K}^0 X). \tag{28}$$

$D \to P_1 P_2$	$C_{P_1}^{D\to P_1 P_2}$	R	$C_R^{D\to P_1 P_2}$
$D_S^+ \to K^0\pi^+$	$V_{cd}^* V_{ud} a_1$	K_0^{*+}	$V_{cs}^* V_{us}$
$D^0 \to \pi^0 \bar{K}^0$	$V_{cs}^* V_{ud} a_2/\sqrt{2}$	$\bar{K}_0^{*0}$	$V_{cs}^* V_{ud} a_2$
$D^0 \to K^-\pi^+$	$V_{cs}^* V_{ud} a_1$	$\bar{K}_0^{*0}$	$V_{cs}^* V_{ud} a_2$
$D^0 \to \eta \bar{K}^0$	$V_{cs}^* V_{ud} a_2(\cos(\theta_\eta)/\sqrt{6} - \sin(\theta_\eta)/\sqrt{3})$	$\bar{K}_0^{*0}$	$V_{cs}^* V_{ud} a_2$
$D^0 \to \eta' \bar{K}^0$	$V_{cs}^* V_{ud} a_2(\sin(\theta_{\eta'})/\sqrt{6} + \cos(\theta_{\eta'})/\sqrt{3})$	$\bar{K}_0^{*0}$	$V_{cs}^* V_{ud} a_2$

Table 1: Table of coefficients in terms of which the invariant amplitudes (27) are expressed. The coefficients for the decays of D^0 meson conjugate to the ones listed in table are obtained by replacing in $D \to \bar{P}_1 \bar{P}_2$ row $V_{cs}^* V_{ud}$ with $V_{cd}^* V_{us}$ and R with $\bar{R}$.

For the description of the decay amplitudes in (27) we use single pole approximation [3, 4] for the form factors describing the D transitions to pseudoscalars

$$F_0^{D_{(s)}\to P}(q^2) = \frac{F_0^{D\to P}(0)}{1 - q^2/m^2_{D^*_{(s)}(0^+)}}. \tag{29}$$

where for the not yet detected scalar D meson mases we take $m_{D^*(0^+)} = 2.47$ GeV, $m_{D_s^*(0^+)} = 2.6$ GeV [4]. The values of form factors at $q^2 = 0$ are $F_0^{D_S\to K}(0) = 0.64$, $F_0^{D\to K}(0) = 0.76$, $F_0^{D\to\pi}(0) = 0.83$, $F_0^{D\to\eta}(0) = 0.68$, $F_0^{D\to\eta'}(0) = 0.66$, taken from [3, 5]. The decay constants are taken from [29, 30] $f_D = 0.21 \pm 0.04$ GeV, $f_{D_S} = 0.24 \pm 0.04$ GeV, $f_\pi = 0.13$ GeV, $f_K = 0.16$ GeV.

The numerical results obtained for the decay widths are given in the first column of the Table 2. We notice that our framework does not lead to the rates which are close enough to the observed decay rates. In this scheme it seems impossible to make the fit with acceptable χ^2. Particularly, the decay rates for $D^0 \to K_S\pi^0$ as well as $D^0 \to \eta' K_S$,$D^0 \to \eta K_S$ do not agree very well with the data, and therefore we conclude that other effects should be taken into account. Following the idea of [4] we include the effects of the final state interaction.

3 The FSI effects in $D \to KP$ decays

It has been pointed out that in the D meson nonleptonic decays one might expect the final state interaction to be of great importance [4, 11, 12, 14]. One way to describe FSI is through unknown phases attached to isospin eigenstates [3, 14]. The phases are then determined from decay widths. The other approach was suggested in [4] assuming that FSI in two-body nonleptonic D meson decays can be described by rescattering through scalar resonance $K_0^*(1950)$ [35] and other resonances belonging to the same nonet. In the description of the rescattering through $K_0^*(1950)$ resonance we account for the $SU(3)$ symmetry breaking by taking the observed masses and decay widths.

The rescattering through a multiplet of resonances mixing different channels is described by [31]

$$M_{N'N}^{FSI} = M_{N'N} - i\frac{\Gamma}{E - m_R + i\Gamma/2}\sum_r a_{N'}^{(r)} a_{N''}^{(r)*} M_{N''N}. \tag{30}$$

Here the summation runs over various mass-degenerated resonances, m_R is the mass of the resonance, $M_{N'N}$ is the invariant amplitude as calculated in (27), while the coefficients $a_N^{(r)}$ describe couplings of N-th state to r-th resonance (the summation over N'' is implicit). Coefficients $a_N^{(r)}$ are orthonormal

$$\sum_N a_N^{(r)*} a_N^{(s)} = \delta_{rs} \qquad \text{and} \qquad \sum_N |a_N^{(r)}|^2 = 1. \tag{31}$$

Since we are interested in the final states with $S \neq 0$ the only resonant state contributing to rescattering we account for is $K_0^*(1950)$. This means there is no summation over r in (30), while the states N (N', N'') are for instance in the case of $\bar{K}_0^{*0}$: $N = \pi^0 \bar{K}^0, K^- \pi^+, \eta \bar{K}^0, \eta' \bar{K}^0$. From the partial decay widths we obtain the coefficients

$$a_{P_1P_2} = \frac{\langle P_1 P_2 |\, \tilde{\mathcal{H}}_{\rm eff}^S \,| K_0^*(1950)\rangle\, p_{P_1P_2}^{1/2}}{\sqrt{\sum_{P_1P_2} \langle P_1 P_2 |\, \tilde{\mathcal{H}}_{\rm eff}^S \,| K_0^*(1950)\rangle^2\, p_{P_1P_2}}}, \tag{32}$$

where $p_{P_1P_2}$ is the momentum of the final particles in the D meson rest frame (20). Here the tilde in $\tilde{\mathcal{H}}_{\rm eff}^S$ denotes the replacement of coupling constants $g_{\ldots} \to \tilde{g_{\ldots}}$ in (22) as now pseudoscalars couple to different resonance, and the replacement $S \to \tilde{S}$, where $\tilde{S}$ denotes the nonet of these higher scalar resonances. Note that our phases differ by a factor 2 in comparison with δ_8 in the equation (3.4) in [4] and are in agreement with [6]. In order to consistently treat $SU(3)$ flavor symmetry breaking we keep the dependence on different masses in $p_{P_1P_2}$, what was not included in analyzes of [4].

Finally the decay width for various decay modes considered reads as

$$\Gamma_{D\to P_1P_2} = \frac{1}{2\pi}\frac{|M_{D\to P_1P_2}^{FSI}|^2}{4m_D^2} p_{P_1P_2}. \tag{33}$$

There are many sources of uncertainty which might arise in our calculation. For example assuming that the nonet symmetry holds both in the case of $K_0^*(1430)$ as in the case of $K_0^*(1950)$, we still have undetermined parameter $f_{K_0^*}$. The additional uncertainties might show up due to the mixing (see (30)) of the four decay channels in which resonance $K_0^*(1950)$ contributes (for instance $D^0 \to \pi^0 \bar{K}^0$, $D^0 \to \pi^+ K^-$, $D^0 \to \eta \bar{K}^0$, $D^0 \to \eta' \bar{K}^0$). If the amplitude for the decay into one of the above intermediate states is poorly known theoretically (e.g. $D^0 \to \eta' \bar{K}^0$ is usually considered to be relatively poorly

known theoretically), then the uncertainty translates into all of the decay modes. The other problem which we encounter in the consideration of the two-body $D_{(s)} \to KX$ decays is that the resonance state $K_0^*(1950)$ is poorly known experimentally with the total decay width $\Gamma = 201 \pm 34 \pm 79$ Mev and branching ratio $\Gamma(K\pi)/\Gamma = 0.52 \pm 0.08 \pm 0.12$. To check how these uncertainties effect calculated decay widths we first vary the $K_0^*(1950) \to K\pi$ partial decay width within experimental bounds, while taking the whole decay width Γ at its average experimental value. This variation results in relaxing the requirement of the exact nonet symmetry for the $K_0^*(1950)$ decay modes. We find that the limits on nonet symmetry breaking parameter are $0.7 < |\tilde{g}_{818}/\tilde{g}_{888}| < 1.4$. However, a relatively small change in $\tilde{g}_{818}/\tilde{g}_{888}$ results in considerable change in decay widths. We found that the best χ^2 fit is realized using the lower bound on $\Gamma(K\pi)/\Gamma$. We present results of this fit in Table 2. With the value $|g_S(K_0^*)| = 3.67\,\mathrm{GeV}$ calculated in previous section, one finds from the fit in the Table 2 $|f_{K_0^*}| = 0.079\,\mathrm{GeV}$. This value is close to results of [25, 20] and close to the value obtained from $K_{\mu 3}^+$.

We have already pointed out that $K_0^*(1950)$ decay parameters have rather large errors. By taking into account uncertainties coming both from $\Gamma(K\pi)/\Gamma$ and the total decay width Γ, we extract only bounds on $-0.13\mathrm{GeV} < f_{K_0^*} < 0.027\mathrm{GeV}$ (where $g_S(K_0^*) = 3.67\,\mathrm{GeV}$ has been chosen with positive sign). Since we cannot determine the sign of $g_S(K_0^*)$, we can only limit $|f_{K_0^*}| < 0.13\,\mathrm{GeV}$. The bound on $|f_{K_0^*}|$ is two standard deviations away from the value obtained from λ_0 in $K_{\mu 3}^0$ decay, while the value that follows from $K_{\mu 3}^+$ decay is well within our limits (see previous section).

Decay	$\Gamma_{\text{no FSI}}$ (10^{-14} GeV)	Γ_{Th} (10^{-14} GeV)	Γ_{Exp} (10^{-14} GeV)
$D_S^+ \to K_S\pi^+$	0.43	0.17	< 1.13
$D^0 \to \pi^0 K_S$	0.44	1.75	1.67 ± 0.18
$D^0 \to \pi^0 K_L$	0.38	1.46	–
$D^0 \to \pi^+ K^-$	6.29	6.41	6.06 ± 0.25
$D^0 \to \eta K_S$	0.13	0.29	0.55 ± 0.09
$D^0 \to \eta K_L$	0.11	0.24	–
$D^0 \to \eta' K_S$	0.19	1.22	1.35 ± 0.22
$D^0 \to \eta' K_L$	0.13	1.02	–
$D^0 \to K^+\pi^-$	0.020	0.013	0.046 ± 0.023

Table 2: The $\Gamma_{\text{no FSI}}$ denotes the calculation of decay widths, where we used the assumption of no final state interaction and fitted the parameter $f_{K_0^*}$ to $f_{K_0^*} g_S(K_0^*) = -0.46\,\mathrm{GeV}^2$. In second column Γ_{Th} are decay widths calculated in case of one resonance rescattering model of final state interaction with $\tilde{g}_{818}/\tilde{g}_{888} = 0.7$, $f_{K_0^*} g_S(K_0^*) = -0.29\,\mathrm{GeV}^2$. Γ_{Exp} denotes experimental values.

One can reach two conclusions from this analysis: first, using the values of

$f_{K_0^*}$ from the existing data one can easily verify that the contribution of the scalar $K_0^*(1430)$ to the decay widths of considered D meson decays is of the same order of magnitude as the spectator terms in factorization approximation. And second, even though fairly far off-shell, the contribution of the scalar projection of the vector meson $K^*(892)$ is still about 20%, due to the large $F_{K^*} = 0.195\mathrm{GeV}^2$ decay constant and consequently the order of magnitude larger $F_{K^*} g_V(K^*)$ compared to scalar $f_{K_0^*} g_S(K_0^*)$ (the last term however being enlarged by $m_D^2/(m_{P_1}^2 - m_{P_2}^2)$ factor in (18)). The point that annihilation terms are not negligible in decay modes considered is best illustrated by notion that χ^2 drops from $\chi^2 = 257$ in case of factorization approach without annihilation terms down to $\chi^2 = 13$ in description with annihilation terms and final state interaction included (Γ_{Th} in Table 2). As seen from the Table 2 also final state interaction contribute significantly to the decay modes considered. With the inclusion of final state interaction the theoretical description of decay channels in Table 2 is in a good agreement with experimental data.

Note however that there still exists some disagreement between the experimental and theoretical value of the $D^0 \to \eta K_S$ decay width for the best χ^2 results (Table 2). This can probably be attributed to the uncertainties in the description of $\eta\eta'$ system that show also in the difficulties with the usual mixing scheme [23]. Note also some discrepancy between the average experimental value and our result in the doubly Cabbibo suppressed $D^0 \to K^+\pi^-$ decay. However a sizable uncertainty in the experimental result suggests that good agreement is hard to be expected.

The uncertainties due to the experimental errors in the input parameters have not been included in the Table 2, but we can roughly estimate that the uncertainties in $f_{D_{(s)}}$,F_{K^*}, $g_V(K^*)$ could result in the errors of about 25%.

4 $\tau \to K P \nu_\tau$ decays

The same matrix element $< KP|j_\mu|0 >$ discussed in the previous section appears in the τ decay. The masses of τ and D are quite close and the bounds on the $K_0^*(1430)$ parameters might be tested in the τ decay spectra and widths. Unfortunately, the data on decay spectrum are not good enough to obtain the information on the $K_0^*(1430)$ parameters [1]. Here we investigate $\tau \to K^-\pi^0\nu_\tau$, $\tau \to \bar{K}^0\pi^-\nu_\tau$, $\tau \to K^-\eta\nu_\tau$ and $\tau \to K^-\eta'\nu_\tau$ decay widths. The first three decay widths have been measured [1], while the decay $\tau \to K^-\eta'\nu_\tau$ has not yet been detected. We start our analyses with the decay width

$$\Gamma = \frac{1}{2E_1}(2\pi)^4\delta^4(p_1 - \cdots - p_n)\sum_{\mathrm{int}}|M_{fi}|^2 \int \prod_{k=2}^{n} \frac{d^3\vec{p}_k}{(2\pi)^3 2E_k}, \tag{34}$$

where

$$M_{fi} = \frac{G_f}{\sqrt{2}} V_{us} J_\mu^L J_H^\mu, \tag{35}$$

and $J_\mu^L = \bar{u}_{\nu_\tau}\gamma_\mu(1-\gamma_5)u_\tau$ in case of τ decays, while J_μ^H is hadronic current (12). Following the procedure in [32] and denoting $y = q^2/m_\tau^2$ we obtain the decay width

$$\Gamma = \int_{y_{\min}}^1 \frac{1}{2}\Gamma_L(1-y)^2\big[4C^3(y)F_+^2(1+2y) + 3C\big(\frac{m_{P_1}^2 - m_{P_2}}{q^2}\big)^2\frac{F_0^2}{y^2}\big]\, dy, \tag{36}$$

where

$$C(q^2) = \frac{p}{\sqrt{q^2}} = \big[\frac{1}{4} - \frac{1}{2}\frac{m_{P_1}^2 + m_{P_2}^2}{q^2} + \frac{(m_{P_1}^2 + m_{P_2}^2)^2}{4q^4}\big]^{1/2}, \tag{37}$$

with p the momentum of outgoing particles in P_1P_2 center of mass frame, $\Gamma_L = G_f^2 m_\tau^5/192\pi^3$ the pure leptonic decay rate, while the lower bound of the integral (36) equals $y_{\min} = (m_{P_1}^2 + m_{P_2})^2/m_\tau^2$.

We present our numerical results for the branching ratios in Table 3, comparing them with the experimental results. The $K_0^*(1430)$ scalar meson does not contribute significantly to these decay rates, as noticed recently by [33]. However, we find out that in the case of $\tau \to K^-\eta'\nu_\tau$ we can expect considerable enhancement of the scalar $K_0^*(1430)$ contribution to the decay width in comparison with the otherwise prevailing $K^*(892)$ vector contribution. This effect has not been taken into account in [34].

In the case of τ decays the final state interaction of the two outgoing pseudoscalar mesons can be neglected. The reason is that the center of mass energy of the pseudoscalar meson pair ranges up to the mass of τ lepton, with the peak of the distribution at the mass of $K^*(892)$, which is below the mass of the $K_0^*(1950)$ resonance. The lower bound $y_{\min}$ of the decay width energy distribution $d\Gamma/dy$ varies with the masses of the final state particles and is for the $\tau \to K^-\eta\nu_\tau$ with 1.04 GeV already above the $K^*(892)$ rest mass, while for the decay $\tau \to K^-\eta'\nu_\tau$ the lower bound is with 1.45 GeV high above the $K^*(892)$ rest mass. For this decay it would be possible that the final state interaction would not be completely negligible, as the upper range of the decay width energy distribution is fairly near to the $K_0^*(1950)$ resonance. However, the bulk of the decay probability lies in the lower part (see Fig. 1), thus we neglect the effect. The final state interaction in $\tau \to K\pi\nu_\tau$ has been approached by the use of chiral perturbation theory and elastic unitarity assumption [16], and found to be rather small.

The other consequence of the different lower bounds $y_{\min}$ for different decay channels is that in the case of $\tau \to K^-\eta'\nu_\tau$ we can expect considerable enhancement of the scalar $K_0^*(1430)$ contribution to the decay width in comparison with

Fig. 1

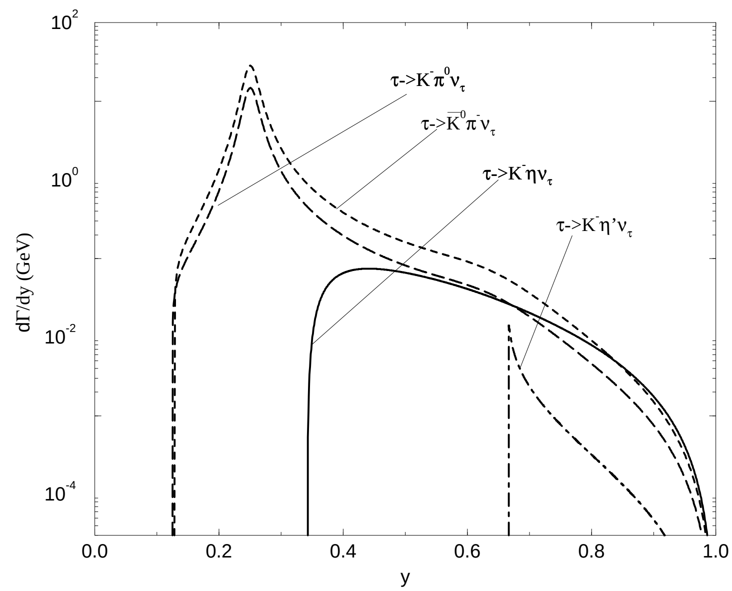

Figure 1: The energy dependence of decay widths for $\tau \to K^-\pi^0\nu_\tau$ in long-dashed, $\tau \to \bar{K}^0\pi^-\nu_\tau$ in dashed, $\tau \to K^-\eta\nu_\tau$ in solid, $\tau \to K^-\eta'\nu_\tau$ in dot-dashed line.

the otherwise prevailing $K^*(892)$ vector contribution. The size of this contribution crucially depends on $f_{K_0^*}$ (see Fig.2). This cannot be extracted inambiguosly from experimental data as discussed in section 2. Thus we use the value of $|f_{K_0^*}| = 0.079\,\mathrm{GeV}$ from the best fit to D decays considered, to evaluate the size of $K_0^*(1430)$ scalar contribution, for which it is an order of magnitude greater than the $K^*(892)$ vector contribution (Table 3). The $\tau \to K^-\eta'\nu_\tau$ decay can thus be viewed as a possible experimental probe for the determination of $f_{K_0^*}$ decay parameter.

Decay	$B_{\mathrm{Th}}(\%)$	$B_{\mathrm{Exp}}(\%)$	scalar (%)
$\tau \to K^-\pi^0\nu_\tau$	0.43	0.52 ± 0.05	2.6
$\tau \to \bar{K}^0\pi^-\nu_\tau$	0.83	0.83 ± 0.08	2.6
$\tau \to K^-\eta\nu_\tau$	0.012	0.027 ± 0.006	6.5
$\tau \to K^-\eta'\nu_\tau$	0.00065	-	90

Table 3: Branching ratios obtained for $f_{K^*}g_S(K_0^*) = -0.29\,\mathrm{GeV}^2$. The scalar denotes the size of scalar $K_0^*(1430)$ contribution to the decay width.

Fig. 2

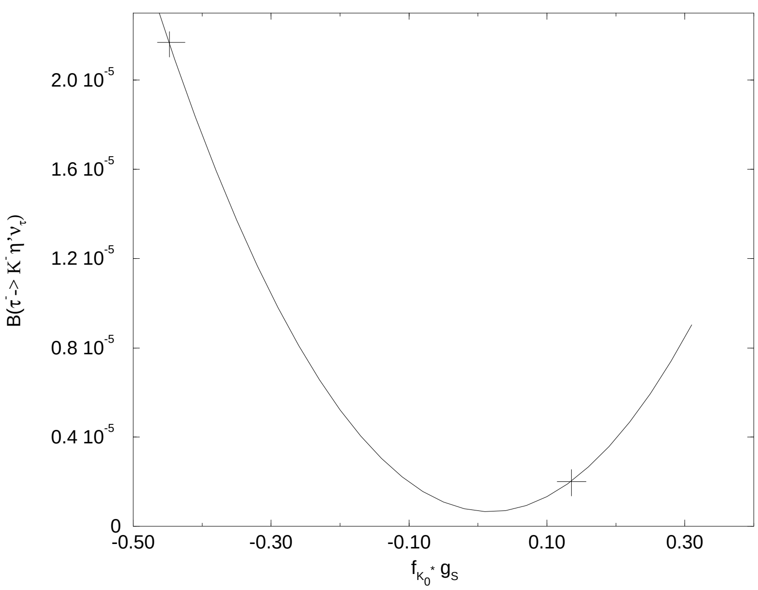

Figure 2: The dependence of calculated $\tau \to K^-\eta'\nu_\tau$ decay width on $f_{K_0^*}g_S$, where the bounds obtained from $D \to KP$, $P = \pi, \eta, \eta'$ decays are denoted with crosses.

5 Summary

We have reinvestigated the $D \to KP$ decays in which the annihilation contribution plays important role. We account as much as possible for the $SU(3)$ flavor symmetry breaking effects by taking directly from the experimental data all relevant parameters. Unfortunately present data cannot find reliably limit on the $K_0^*(1430)$ decay coupling constant, therefore we kept it as a free parameter.

In the treatment of scalar form factor defined in the $< KP|j_\mu|0 >$ matrix element, we included the contribution of the scalar projection of the vector meson $K^*(892)$ resonance. We found that the contribution of the $K^*(892)$ resonance in the annihilation amplitude is about 20% of the one coming from the scalar meson $K_0^*(1430)$ resonance.

The inclusion of the final state interaction through the rescattering via $K_0^*(1950)$ resonance in these decays improves the agreement with the data. We then estimated bounds on the $K_0^*(1430)$ decay coupling $|f_{K_0^*}| < 0.13\,\mathrm{GeV}$. The obtained bounds are in agreement with the experimental results obtained in $K^+_{\mu 3}$ decay. Although many other effects might be present in D^0 decays, like nonfactorizable contributions, we try to understand the role of scalar resonances in the annihilating contribution within this framework.

Our analyses of the $\tau \to KP\nu_\tau$, $P = \pi, \eta$, decay spectra widths shows rather

small contribution of the scalar $K_0^*(1430)$ resonance. However, we notice its significant presence (about 90% in the rate) in the $\tau \to K\eta'\nu_\tau$ decay. The type of the final state interaction analyzed in D nonleptonic decays was not important in τ decays.

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